# A Theoretical Study on the Photochemical Reaction of Carbon Suboxide with Ethylene 

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#### Abstract

The photochemical reaction of carbon suboxide with ethylene is studied theoretically in terms of state correlation diagrams and an ab initio calculation. In light of the calculated results, the reaction is suggested to consist of the following processes. Starting from the second excited state of carbon suboxide, the ground-state carbonylcarbene (triplet) is yielded through the bent dissociation path. The carbonylcarbene adds to ethylene in the "bent-in-plane" path, producing cyclopropylideneketene. The cyclopropylideneketene is decomposed through the bent-in-plane path into cyclopropylidene and carbon monoxide. Finally, the cyclopropylidene collapses to form the ground-state allene. The mechanisms of these reactions are elucidated by the mode of orbital interaction.


## I. Introduction

Carbon suboxide $\left(\mathrm{C}_{3} \mathrm{O}_{2}\right)$ is known to undergo a photochemical reaction with olefin. ${ }^{2}$ The single carbon atom of $\mathrm{C}_{3} \mathrm{O}_{2}$ is inserted into the olefinic double bond, yielding an allene $\left(\mathrm{C}_{3} \mathrm{H}_{4}\right)$ and two molecules of carbon monoxide (CO). The overall reaction of $\mathrm{C}_{3} \mathrm{O}_{2}$ with ethylene $\left(\mathrm{C}_{2} \mathrm{H}_{4}\right)$ is given as follows:

$$
\mathrm{C}_{3} \mathrm{O}_{2}+\mathrm{C}_{2} \mathrm{H}_{4} \rightarrow \mathrm{C}_{3} \mathrm{H}_{4}+2 \mathrm{CO}
$$

When $\mathrm{C}_{3} \mathrm{O}_{2}$ is labeled in the central position ( $\mathrm{C}^{*}$ ), the newly inserted atom is found in the central position of $\mathrm{C}_{3} \mathrm{H}_{4} \cdot{ }^{3}$ The reactive intermediate is probably either a carbonylcarbene $\left(\mathrm{C}_{2} \mathrm{O}\right)$ molecule or a free carbon atom. Energy consideration, based on the estimated heat of formation for $\mathrm{C}_{3} \mathrm{O}_{2}$, rules out the possibility of a free carbon atom. ${ }^{4}$ The general reaction scheme involving $\mathrm{C}_{2} \mathrm{O}$ is represented as follows.

$$
\begin{gather*}
\mathrm{O}=\mathrm{C}=\mathrm{C}^{*}=\mathrm{C}=\mathrm{O} \rightarrow \mathrm{C}^{*}=\mathrm{C}=\mathrm{O}+\mathrm{C}=\mathrm{O}  \tag{1}\\
\mathrm{C}^{*}=\mathrm{C}=\mathrm{O}+\mathrm{C}_{2} \mathrm{H}_{4} \rightarrow \mathrm{CH}_{2}=\mathrm{C}^{*}=\mathrm{CH}_{2}+\mathrm{CO} \tag{2}
\end{gather*}
$$

Williamson and Bayes pointed out that the photolysis (at 3000 and $2500 \AA$ ) shown in eq 1 produced two different intermediates which were thought to be $\mathrm{C}_{2} \mathrm{O}\left({ }^{3} \Sigma^{-}\right)$and $\mathrm{C}_{2} \mathrm{O}\left({ }^{1} \Delta\right) .{ }^{5}$ When they react with olefin as shown in eq 2 , $\mathrm{C}_{2} \mathrm{O}\left({ }^{3} \Sigma^{-}\right)$displays an electrophilic character but $\mathrm{C}_{2} \mathrm{O}\left({ }^{1} \Delta\right)$ is quite indiscriminate. In the reaction of $\mathrm{C}_{2} \mathrm{O}\left({ }^{3} \Sigma^{-}\right)$with olefin, a second intermediate was postulated by Willis and Bayes, ${ }^{6}$ While the identification was not certain, they suggested that it was either cyclopropylidene (CP) or cyclopropylideneketene (CK).


CK


CP

Although it is well accepted that the first step of the photoreaction of $\mathrm{C}_{3} \mathrm{O}_{2}$ with olefin is the dissociation of $\mathrm{C}_{3} \mathrm{O}_{2}$ and the pathway of $\mathrm{C}_{3} \mathrm{O}_{2}\left({ }^{3} \Delta_{u}\right) \rightarrow \mathrm{C}_{2} \mathrm{O}\left({ }^{3} \Sigma^{-}\right)+\mathrm{CO}$ has been traced theoretically, ${ }^{7}$ the reaction mechanism of the second step represented by eq 2 still involves some doubt. In the present work, the path and the mechanism of the reaction of $\mathrm{C}_{2} \mathrm{O}$ with olefin are explored. While the search for the dissociation path
of eq 1 is rather straightforward by use of the energy-optimization procedure, the reaction of eq 2 must be analyzed carefully. This is because the reaction may involve several processes such as additions and dissociations and one or more intermediates (e.g., CP and CK). In order to analyze the mechanism of reaction 2 , the state correlation diagrams for various processes are drawn systematically and these reaction paths are qualitatively predicted. Based on this prediction, the mini-mum-energy paths are traced by an ab initio calculation. In this work the reaction starting from the ${ }^{3} \Sigma^{-}$state of $\mathrm{C}_{2} \mathrm{O}$ is dealt with, since the calculation for the ${ }^{1} \Delta$ state of $\mathrm{C}_{2} \mathrm{O}$ is technically somewhat difficult.

## II. Consideration of Reaction Paths Based on the State Correlation Diagram

To draw state correlation diagrams, the possible reaction paths involved in eq 2 are investigated and listed in*Table I. In this table, it is assumed that the reaction takes place via an intermediate similar to CP or CK, which is based on the suggestion by Willis and Bayes. ${ }^{6}$ While the reaction path of eq 1 is obviously either linear or bent, that of eq 2 is very complicated. For easier understanding, the possible paths are given in Figure 1, and the most favorable path for each process is selected. The way of examining the reaction path by the state correlation diagrams is the same as that used previously. ${ }^{8}$

Photodissociation of $\mathrm{C}_{3} \mathrm{O}_{2}$ in Equation 1. Although the minimum-energy path of this reaction is already followed, ${ }^{7}$ let us review it in terms of the correlation diagram. As Table I shows, there are two types of path in the photodissociation of $\mathrm{C}_{3} \mathrm{O}_{2}$. One is the least motion path which is the linear fragmentation of $C_{\infty h}$ symmetry. The other is the bent path of $C_{s}$ symmetry. While one of the dissociation products, $\mathrm{C}_{2} \mathrm{O}$, is expected to have several low-lying excited states, the other, CO , may be fixed to its ground state ( ${ }^{1} \Sigma^{+}$). The state correlation diagram for the linear fragmentation is given in Figure 2a. In this figure, the number displayed at both edges denotes the assignment of 34 electrons in a given configuration to each orbital ( $\sigma, \pi_{x}$, and $\pi_{y}$ ). It is found that this path is symmetry disfavored in the ground state ( ${ }^{1} \Sigma_{\mathrm{g}}{ }^{+}$) and some low-lying excited states ( ${ }^{3} \Sigma_{u}{ }^{+}, 1,3 \Delta_{\mathfrak{u}}$, and ${ }^{1,3} \Sigma_{u}{ }^{-}$) owing to the avoided crossing. Figure 2 b shows the correlation diagram for the bent path in which the $C_{\infty h}$ symmetry imposed on the reaction system is reduced to $C_{s}$ symmetry. As Figure 2 b shows, the ground-state dissociation is still energetically unfavorable in the bent path. The dissociation in some excited states ( ${ }^{3} \Delta_{u}$ and ${ }^{1,3} \Sigma_{\mathrm{u}}{ }^{-}$), however, becomes symmetry allowed in this pathway,

Table I, List of Various Types of Reaction Paths Displayed in Figure 1

| $\begin{gathered} \text { reaction } \\ \text { eq } \\ \hline \end{gathered}$ | no. | reaction scheme | type of reaction | pathways (symmetry of the system) |
| :---: | :---: | :---: | :---: | :---: |
| 1 | (1) | $\mathrm{C}_{3} \mathrm{O}_{2} \rightarrow \mathrm{C}_{2} \mathrm{O}+\mathrm{CO}$ | photodissociation of carbon suboxide addition of carbonyl- | $\begin{aligned} & \text { linear }\left(C_{\infty h}\right) \\ & \text { bent }\left(C_{s}\right) \end{aligned}$ |
|  | (2) | $\mathrm{C}_{2} \mathrm{O}+\mathrm{C}_{2} \mathrm{H}_{4} \rightarrow \mathrm{C}_{4} \mathrm{H}_{4} \mathrm{O}$ | carbene to ethylene to form CK | I least motion ( $C_{2 v}$ ) <br> II bent-in-plane ( $C_{s}$ ) <br> ( $X-Z$ symmetry plane $)^{a}$ <br> III bent-out-of-plane ( $C_{\mathrm{s}}{ }^{\prime}$ ) <br> $(Y-Z \text { symmetry plane })^{b}$ |
|  | (3) | $\mathrm{C}_{4} \mathrm{H}_{4} \mathrm{O} \rightarrow \mathrm{C}_{3} \mathrm{H}_{4}+\mathrm{CO}$ | concerted (ring opening and elimination of CO ) |  |
| 2 | (4) | $\mathrm{C}_{4} \mathrm{H}_{4} \mathrm{O} \rightarrow \mathrm{C}_{3} \mathrm{H}_{4}+\mathrm{CO}$ | dissociation of CK to CP and CO |  |
|  | (5) | $\mathrm{C}_{3} \mathrm{H}_{4} \rightarrow \mathrm{C}_{3} \mathrm{H}_{4}$ | ring opening of CP to form planar allene | IV conrotatory $\left(C_{2}\right)$ <br> V disrotatory $\left(C_{s}\right)$ <br> VI asymmetric ( $C_{1}$ ) |
|  | (6) | $\mathrm{C}_{3} \mathrm{H}_{4} \rightarrow \mathrm{C}_{3} \mathrm{H}_{4}$ | intersystem crossing ( $\mathrm{CP} \rightarrow$ ground-state allene) without passing the excited-state allene intersystem crossing to form the ground-state allene |  |
|  | (7) | $\mathrm{C}_{3} \mathrm{H}_{4} \rightarrow \mathrm{C}_{3} \mathrm{H}_{4}$ |  |  |

[^0]

Figure 1, Possible mechanism for the reaction of eq 2. The routes in this figure are defined as the "reaction schemes" in Table I. In the right side of the figure, $T_{1}$ allene is assumed to have the planar and linear geometry, while its most stable structure is planar and bent. This assumption is needed so as to draw the state correlation diagram of the $C_{2 v}$ symmetry in Figure 3a. Since the energy difference between linear and bent structures is small ( $\sim 8$ $\mathrm{kcal} / \mathrm{mol}$ ), the linear $\mathrm{T}_{1}$ allene is used throughout the text.

It is found that the second excited state of $\mathrm{C}_{3} \mathrm{O}_{2}\left({ }^{3} \Delta_{u}\right)$ yields the ground-state $\mathrm{C}_{2} \mathrm{O}\left({ }^{3} \Sigma^{-}\right)$and CO (the bold line in Figure 2 b ). The reaction path discussed here is in line with that of the previous calculation. ${ }^{7}$ Thus, the "ground-state" $\mathrm{C}_{2} \mathrm{O}$ is generated through the bent path.
Reaction of $\mathrm{C}_{2} \mathrm{O}$ with $\mathrm{C}_{2} \mathrm{H}_{4}$ in Equation 2. Schemes (2)-I $\rightarrow$ (3)-I and (2)-I $\rightarrow$ (4)-I. Here, (2)-I denotes path (2) with $C_{2 v}$ symmetry (see Table I). In the first process, the product allene is assumed to be of $C_{20}$ symmetry. ${ }^{9}$ Figure 3 a shows the state correlation diagram for this process. Our attention is focused on the ${ }^{3} \mathrm{~A}_{2}$ state, because the ground-state $\mathrm{C}_{2} \mathrm{O}\left({ }^{3} \Sigma^{-}\right)$is involved in this state. It is found that in the ${ }^{3} \mathrm{~A}_{2}$ state the scheme (2)-I is a symmetry-disfavored path because of the avoided crossing, while the scheme (3)-I of ${ }^{3} \mathrm{CK}$ (i.e., triplet CK) $\rightarrow$ ${ }^{3} \mathrm{C}_{3} \mathrm{H}_{4}+\mathrm{CO}$ is a symmetry-allowed path. From this figure, the reaction path of the scheme (2)-I maintaining $C_{2 v}$ symmetry is unfavorable and is ruled out. However, the schemes (3)-l, (3)-II, and (3)-III, the latter two of which have lower
symmetry than $C_{2 v}$, are found to be the symmetry-allowed paths. Figure 3 b shows that the scheme (4)-I of the second process is symmetry disfavored.
Schemes (2)-II $\rightarrow$ (4)-II. In this process, the intermediacy of ${ }^{3} \mathrm{CK}$ and ${ }^{3} \mathrm{CP}$ is assumed. Figure 4 shows the state correlation diagram of $C_{s}$ symmetry for the process (2)-II $\rightarrow$ (4)-II, which is based on the correlation diagram of $C_{2 v}$ symmetry exhibited in Figure 3. The curve connecting the lowest ${ }^{3} \mathrm{~A}$ states corresponds to the reaction of $\mathrm{C}_{2} \mathrm{O}\left({ }^{3} \Sigma^{-}\right)+\mathrm{C}_{2} \mathrm{H}_{4} \rightarrow{ }^{3} \mathrm{CP}$ +CO . As Figure 4 shows, schemes (2)-II $\rightarrow$ (4)-II give a symmetry-allowed path (bold line).
Schemes (2)-III $\rightarrow$ (4)-III. In this process, the intermediacy of ${ }^{3} \mathrm{CK}$ and ${ }^{3} \mathrm{CP}$ is again assumed. The correlation diagram of $C_{s}$ ' symmetry for the schemes (2)-III and (4)-III is pictured in Figure 5. The curve connecting the ${ }^{3} \mathrm{~A}$ states corresponds to the reaction of $\mathrm{C}_{2} \mathrm{O}\left({ }^{3} \Sigma^{-}\right)+\mathrm{C}_{2} \mathrm{H}_{4} \rightarrow{ }^{3} \mathrm{CP}+\mathrm{CO}$. In this state, the scheme (2)-III is given a symmetry-disfavored path owing to the avoided crossing and the scheme (4)-III is unfa-
vorable because the ${ }^{3} \mathrm{~A}$ state of CP is the high-lying state. Thus, the schemes (2)-III and (4)-III are ruled out from the possible reaction paths.
Schemes (5)-IV, V, and VI. In the ring opening of ${ }^{3} \mathrm{CP}$, the product ${ }^{3} \mathrm{C}_{3} \mathrm{H}_{4}$ is assumed to have a bent planar structure $\left(C_{2 v}\right)$, which is the same as for Figure 3a. Figure 6a shows the correlation diagram of $C_{2}$ symmetry for the conrotatory ring opening. Our attention is focused on the curve connecting the ${ }^{3}$ A state, which originates from the reaction of $\mathrm{C}_{2} \mathrm{O}\left({ }^{3} \Sigma^{-}\right)+$ $\mathrm{C}_{2} \mathrm{H}_{4}$. As Figure 6a shows, the conrotatory ring opening is unfavorable because the ${ }^{3}$ A state of the product $\mathrm{C}_{3} \mathrm{H}_{4}$ is the high-lying excited one. Figure 6 b shows the correlation diagram of $C_{s}$ symmetry for the disrotatory ring opening. The ${ }^{3} \mathrm{~S}$ state comes from the reaction of $\mathrm{C}_{2} \mathrm{O}\left({ }^{3} \Sigma^{-}\right)+\mathrm{C}_{2} \mathrm{H}_{4}$. As Figure 6 b shows, the scheme (5)-V for the ${ }^{3} \mathrm{~S}$ state is an energetically unfavorable path. From these results, the ring opening of ${ }^{3} \mathrm{CP}$ is considered to proceed along the asymmetric path [scheme (5)-VI]. However, the results of ${ }^{3} \mathrm{CP} \rightarrow{ }^{3} \mathrm{C}_{3} \mathrm{H}_{4}$ calculated by Pasto et al. and Dillon and Underwood do not agree with this discussion. ${ }^{10}$ In our analysis, the schemes (5)-IV and V are ruled out from the possible reaction paths.

Let us summarize the results obtained so far from the correlation diagrams. Some of the reaction schemes of eq 2 can be excluded either owing to the symmetry restriction or owing to the high-lying state energy and are indicated by the broken lines in Figure 1. Then the possible reaction schemes are reduced to the following five processes:

$$
\begin{gathered}
(2)-\mathrm{II} \rightarrow(4)-\mathrm{II} \rightarrow(5)-\mathrm{VI} \rightarrow(7) \\
(2)-\mathrm{II} \rightarrow(4)-\mathrm{II} \rightarrow(6) \\
(2)-\mathrm{HI} \rightarrow(3)-\mathrm{I} \rightarrow(7) \\
(2)-\mathrm{II} \rightarrow(3)-\mathrm{HI} \rightarrow(7) \\
(2)-\mathrm{II} \rightarrow(3)-\mathrm{III} \rightarrow(7)
\end{gathered}
$$

While the first step of eq 2 is expected to be the addition of $\mathrm{C}_{2} \mathrm{O}$ into $\mathrm{C}_{2} \mathrm{H}_{4}$ through the bent-in-plane path, further information on the probable path must be obtained from a quantitative analysis. Also, the intermediacy of CK and/or CP which is assumed in the qualitative discussion must be checked energetically. In the next section, the five possible reaction schemes are compared by ab initio calculations and the most probable process is sought.

## III. Results of the Minimum-Energy Path

The minimum-energy paths ${ }^{11}$ for eq 2 are examined by the optimization procedure of the total energy ( $E_{\mathrm{T}}$ ). The $E_{\mathrm{T}}$ of the reaction system of the triplet spin multiplicity is calculated by the GRHF method ${ }^{12}$ with the STO-3G minimal basis set. ${ }^{13}$

Scheme (2)-II. This scheme is found to be symmetry favored in the previous section. The geometrical parameters to be optimized are shown in Figure 7a. As the result of calculations, the drawings of the optimized geometries and the potentialenergy profile along the minimum-energy path are shown in Figures 8 a and 8 b , respectively. ${ }^{14}$ Figure 8 a shows that in the scheme (2)-Il the $\mathrm{C}_{1} \cdots \mathrm{C}_{6}$ bond is formed primarily and then the three-membered ring is formed. The energy profile shows a local minimum at $R \simeq 1.9 \AA$ which corresponds to a biradical intermediate. This intermediacy is found also in the photodissociation of cyclopropanone. ${ }^{8 \mathrm{~b}}$ It should be noted that in the energy profile of Figure 8 b this reaction has no energy barrier.

Schemes (3)-I,II and (4)-II. These four schemes appear in the second step of eq 2 (the paths with full and bold lines in Figure 1). Since the most stable structure of ${ }^{3} \mathrm{CK}$ is the bent-in-plane type of $C_{s}$ symmetry, ${ }^{3} \mathrm{CK}$ of $C_{2 v}$ symmetry and that of $C_{s}{ }^{\prime}$ symmetry have no minima. These geometries ( $C_{2 v}$ and $C_{s}{ }^{\prime}$ ) are merely models for comparison and such energetic insta-


Figure 2, (a) The state correlation diagram for the linear fragmentation of $\mathrm{C}_{3} \mathrm{O}_{2}$ in eq 1. At both edges, the sets of numbers represent the number of electrons assigned to each MO. (b) The state correlation diagram for the bent dissociation of $\mathrm{C}_{3} \mathrm{O}_{2}$ in eq 1 .
bility makes the deformation ( $C_{s} \rightarrow C_{2 p}$ or $C_{s}$ ) of CK improbable, which is indicated by two crosses attached to the vertical lines in Figure 1. Thus, the schemes (3)-I and (3)-11I via the CK of $C_{2 v}$ or $C_{s}{ }^{\prime}$ may be ruled out. Now the possible schemes of the decarbonylation are limited to the schemes (3)-II and (4)-1l. In order to decide which scheme is more fa-
(a) $\mathrm{C}_{2 \mathrm{~V}} \quad \mathrm{C}_{2} \mathrm{O}+\mathrm{C}_{2} \mathrm{H}_{4}($ Gr. $) \rightarrow \mathrm{CK} \rightarrow$ allene $+\mathrm{CO}($ Gr. $)$

(b) $\quad \mathrm{C}_{2 \mathrm{v}} \quad \mathrm{CK} \rightarrow \mathrm{CP}+\mathrm{CO}(\mathrm{Gr}$.


Figure 3, (a) The state correlation diagram of $C_{2 v}$ symmetry for the schemes (2)-I $\rightarrow$ (3)-1 in eq 2. (b) The state correlation diagram of $C_{2 v}$ symmetry for the scheme (4)-I in eq 2. $\mathrm{C}_{2} \mathrm{O}^{3} \Sigma^{-}$) has the electron configuration ... $\left(7 \mathrm{a}_{1}\right)^{2}\left(2 \mathrm{~b}_{1}\right)^{1}\left(2 \mathrm{~b}_{2}\right)^{1}\left(3 \mathrm{~b}_{1}\right)^{0}\left(3 \mathrm{~b}_{2}\right)^{0}\left(8 \mathrm{a}_{1}\right)^{0} \ldots$. The ground-state CK has the electron configuration ... $\left(\mathrm{Ia}_{2}\right)^{2}\left(10 \mathrm{a}_{1}\right)^{2}\left(4 \mathrm{~b}_{1}\right)^{2}\left(3 \mathrm{~b}_{2}\right)^{2}\left(5 \mathrm{~b}_{1}\right)^{0}\left(4 \mathrm{~b}_{2}\right)^{0}\left(\mathrm{II} \mathrm{a}_{1}\right)^{0} \ldots$. The ground configuration of CP is $\ldots\left(\mathrm{Ia}_{2}\right)^{2}\left(3 \mathrm{~b}_{1}\right)^{2}\left(6 \mathrm{a}_{1}\right)^{2}\left(2 \mathrm{~b}_{2}\right)^{0}\left(4 \mathrm{~b}_{1}\right)^{0}$ $\ldots$. The ground configuration of linear allene is ... $\left(1 a_{2}\right)^{2}\left(5 a_{1}\right)^{2}\left(4 b_{1}\right)^{2}\left(2 b_{2}\right)^{0}\left(6 a_{1}\right)^{0}\left(7 a_{1}\right)^{0} \ldots$.


Figure 4, The state correlation diagram of $C_{s}$ symmetry for the schemes (2)-II $\rightarrow$ (4)-II in eq 2 . The $C_{2 t}$ symmetry imposed on the reacting system is reduced to $C_{s}$ symmetry.
$C_{s}$ ' (Bent-out-of-Plane)


(2) - III
(4) - III


Figure 5, The state correlation diagram of $C_{s}{ }^{\prime}$ symmetry for the schemes (2)-III $\rightarrow$ (4)-III in eq 2. The $C_{2 v}$ symmetry of the system is converted to $C_{s}{ }^{\prime}$ symmetry.
vorable, the minimum-energy path of the scheme (4)-11 is first examined. The geometrical parameters are shown in Figure 7b. Here, the CP part of CK (i.e., three-membered ring) is kept frozen, because the geometry of the CP fragment is similar to that of the isolated CP according to our preliminary calculation. Figures 9 a and 9 b show drawings of the decarbonylation pathway and the potential-energy profile along the mini-mum-energy path. As Figure 9a shows, the decarbonylation $\left({ }^{3} \mathrm{CK} \rightarrow{ }^{3} \mathrm{CP}+\mathrm{CO}\right)$ takes the bent-in-plane path. This path is similar to that of the dissociation of the $\left(\pi, \pi^{*}\right)$ ketene. ${ }^{8 a}$

Next, to see whether the path of the scheme (3)-II is energetically favorable or not, $E_{\mathrm{T}}$ is calculated for the geometry with the elongated $\mathrm{C}_{1}-\mathrm{C}_{2}$ and $\mathrm{C}_{6}-\mathrm{C}_{7}$ bonds. The calculation for this "concerted" mechanism gives a much more unstable system than that of the scheme (4)-II (only the $\mathrm{C}_{1}-\mathrm{C}_{2}$ scission given). Thus, the possibility that the $\left(\mathrm{C}_{1}-\mathrm{C}_{2}-\mathrm{C}_{3}\right)$ ring opening and the dissociation of the $\mathrm{C}_{6}-\mathrm{C}_{7}$ bond take place simultaneously and cooperatively is more unlikely than that of the stepwise reaction. That is, the path of the scheme (4)-II is more favorable than that of the scheme (3)-II, which suggests that the reaction of eq 2 does pass the intermediate CP .

When the energetic superiority or inferiority is compared, two probable processes of eq 2 among the five symmetry-favored ones can be considered. One is the process (2)-11 $\rightarrow$ (4)-II $\rightarrow$ (5)-IV $\rightarrow$ (7) and the other is the process (2)-II $\rightarrow$ (4)-II $\rightarrow$ (6). Since the $E_{T}$ of ${ }^{3} \mathrm{CP}$ is lower than that of ${ }^{1} \mathrm{CP}^{9}$ and the $E_{\mathrm{T}}$ of the planar ${ }^{3} \mathrm{C}_{3} \mathrm{H}_{4}$ is appreciably high, the ground-state $\mathrm{C}_{3} \mathrm{H}_{4}$ may be yielded from ${ }^{3} \mathrm{CP}$ by the intersystem crossing (isc). In this regard, the latter process is believed to be more likely than the former.

From the results of correlation diagrams and the calculation of $E_{\mathrm{T}}$, the photochemical reaction of $\mathrm{C}_{3} \mathrm{O}_{2}$ with $\mathrm{C}_{2} \mathrm{H}_{4}$ is shown most likely to proceed through reacting steps (2)-II $\rightarrow$ (4)-II $\rightarrow(6)$. Here, the question of which step is rate determining should be answered. The energy barrier for the photodissociation of $\mathrm{C}_{3} \mathrm{O}_{2}$ in eq 1 is calculated to be about $18 \mathrm{kcal} / \mathrm{mol},{ }^{7}$ and that of the decarbonylation of ${ }^{3} \mathrm{CK}$ in Figure 9 b is about $35 \mathrm{kcal} / \mathrm{mol}$. Pasto et al. reported that the ring-opening step of ${ }^{3} \mathrm{CP}$ has a $19 \mathrm{kcal} / \mathrm{mol}$ barrier with the $4-31 \mathrm{G}$ basis set and that of ${ }^{1} \mathrm{CP}$ has $18 \mathrm{kcal} / \mathrm{mol} .{ }^{9}$ Comparing these energy values together with a small energy barrier of the scheme (2)-II, one may suggest that the rate-determining step of the overall reaction ( $\left.\mathrm{C}_{3} \mathrm{O}_{2}+\mathrm{C}_{2} \mathrm{H}_{4} \rightarrow \mathrm{C}_{3} \mathrm{H}_{4}+2 \mathrm{CO}\right)$ is the decarbonylation of ${ }^{3} \mathrm{CK}$, i.e., the scheme (4)-II.
(a) $\mathrm{C}_{2}$ (Conrotatory)

(b) $\mathrm{C}_{5}$ (Disrotatory)



Figure 6. (a) The state correlation diagram of $C_{2}$ symmetry for the scheme (5)-IV in eq 2. (b) The state correlation diagram of $C_{s}$ symmetry for the scheme (5)-V in eq 2 . The bent planar allene in the ground state has the electron configuration ... $\left(4 \mathrm{~b}_{1}\right)^{2}\left(1 \mathrm{~b}_{2}\right)^{2}\left(1 \mathrm{a}_{2}\right)^{2}\left(6 \mathrm{a}_{1}\right)^{0}\left(2 \mathrm{~b}_{2}\right)^{0}\left(7 \mathrm{a}_{1}\right)^{0} \ldots$

## IV. Discussion Based on Orbital Interaction

The results of the correlation diagrams and the calculation of the $E_{\mathrm{T}}$ suggest the favorable reaction schemes mentioned above. In this section, they are discussed in terms of the mode of orbital interaction. ${ }^{15}$


Figure 7, (a) Definition of the geometrical parameters to be optimized for obtaining the minimum-energy path for the scheme (2)-II. (b) The same for the scheme (4)-II.

Photodissociation of $\mathrm{C}_{3} \mathrm{O}_{2}$ in Equation 1. The motion of the leaving CO during the bent dissociation which is obtained by the energy optimization can be interpreted in terms of the orbital interaction scheme. The electron delocalization indicated by arrows in the following figure makes the reacting system stabilized. That is, the orientation of CO toward $\mathrm{C}_{2} \mathrm{O}$ given as

an energy-minimum point is in accord with that of the maximum overlapping interactions of the particular orbitals indicated in the following diagram. These particular orbitals are the doubly occupied (DO) MO, the highest occupied (HO) MO, the singly occupied (SO) MO, and the lowest unoccupied (LU) MO. Each orbital interaction between them is called a charge transfer (CT) interaction separately.

Reaction of $\mathrm{C}_{2} \mathrm{O}$ with $\mathrm{C}_{2} \mathrm{H}_{4}$ in Equation 2. In the previous section, the reaction of $\mathrm{C}_{2} \mathrm{O}$ with $\mathrm{C}_{2} \mathrm{H}_{4}$ is shown to involve the
schemes (2)-II, (4)-II, and (6), where the former two schemes are traced by the ab initio calculations. The reaction mechanisms of the schemes (2)-Il and (4)-II are now investigated.

Scheme (2)-II. In this reaction it is shown that the $\mathrm{C}_{1} \cdots \mathrm{C}_{6}$ bond is formed primarily and then the three-membered ring is formed. This reaction process is explained by the CT and exchange ( $E_{\mathrm{K}}$ ) interactions. In the process of the $\mathrm{C}_{1} \cdots \mathrm{C}_{6}$ bond formation, three types of $\mathrm{CT}\left(\mathrm{CT}_{1}, \mathrm{CT}_{2}\right.$, and $\left.\mathrm{CT}_{3}\right)$ interactions given in the left side of the following figure contribute to the formation of the new bond. However, in the $C_{1} \cdots C_{7}$ region, the

$E_{\mathrm{K}}$ repulsion (dotted arrow) between the DOMO of $\mathrm{C}_{2} \mathrm{O}$ and the HOMO of $\mathrm{C}_{2} \mathrm{H}_{4}$ interferes with the $\mathrm{C}_{1} \cdots \mathrm{C}_{7}$ bond formation, overcoming the attractive effect of $\mathrm{CT}_{2}$. Consequently, the biradical species is formed. Next, $\mathrm{CT}_{4}$ and $\mathrm{CT}_{5}$ shown in the middle of the figure contribute to the formation of the three-membered ring.

The two unpaired electrons with parallel spins involved in the biradical intermediate in this process have an interesting behavior. While one of these unpaired electrons is localized almost at the $2 \mathrm{p}_{y}$ atomic orbital of the $\mathrm{C}_{1}$ atom during the reaction of the scheme (2)-II, the other electron in the $X-Z$ plane moves from the $\mathrm{C}_{1}$ atom of $\mathrm{C}_{2} \mathrm{O}$ to the $\mathrm{C}_{7}$ atom of $\mathrm{C}_{2} \mathrm{H}_{4}$ and then from the $\mathrm{C}_{7}$ atom to the $\mathrm{C}_{2}$ atom of the former. This



Figure 8, (a) The drawing of the optimized geometry for the scheme (2)-II. $O, \oplus$, and $\ominus$ denote the carbon, oxygen, and hydrogen atoms, respectively (b) The potential-energy profile along the minimum-energy path for the scheme (2)-II.


Figure 9, (a) The drawing of the optimized geometry for the scheme (4)-11. (b) The potential-energy profile along the minimum-energy path for the scheme (4)-II.
movement of the latter electron is explicable by the mode of orbital interaction. ${ }^{16}$ The first movement $\left(C_{1} \rightarrow C_{7}\right)$ is brought about by the interaction of the SOMO of $\mathrm{C}_{2} \mathrm{O}$ with the $\pi$ and $\pi^{*}$ (HOMO and LUMO) orbitals of $\mathrm{C}_{2} \mathrm{H}_{4}$. The interaction of the SOMO with the $\pi$ and $\pi^{*}$ orbitals gives three orbitals $\left(\psi_{1}, \psi_{2}\right.$, and $\psi_{3}$ ), where $\psi_{2}$ is the unpaired-electron orbital of

the reacting system. The $\psi_{2}$ orbital is represented as follows. ${ }^{16}$

$$
\psi_{2}=\mathrm{SOMO}-\pi+\pi^{*}
$$

The mixing of the $\pi$ and $\pi^{*}$ through the SOMO contributes to the spin transfer in the $\mathrm{C}_{1} \rightarrow \mathrm{C}_{7}$ direction.

The movement ( $\mathrm{C}_{7} \rightarrow \mathrm{C}_{2}$ ) is also explicable by the interaction of the orbitals (SOMO of $\mathrm{C}_{7}$, DOMO and LUMO of the $\mathrm{C}_{2} \mathrm{O}$ part in the biradical species) spreading over the region in which the new $C_{1} \cdots C_{7}$ bond is formed. The interaction of the SOMO with the DOMO and the LUMO gives three orbitals ( $\phi_{1}, \phi_{2}$, and $\phi_{3}$ ) where the $\phi_{2}$ orbital becomes the unpairedelectron orbital of the biradical intermediate. The $\phi_{2}$ orbital is also represented as follows:

$$
\phi_{2}=\mathrm{SOMO}-\mathrm{DOMO}+\mathrm{LUMO}
$$

Therefore, the mixing of the DOMO and the LUMO contributes to the ( $\mathrm{C}_{7} \rightarrow \mathrm{C}_{2}$ ) spin transfer.

Scheme (4)-II. Since the reaction of this scheme is shown to be the rate-determining step of the overall reaction, it is informative to clarify its reaction mechanism. The motion of the

leaving CO in the decarbonylation obtained by the energyoptimized procedure is explained by the following CT interactions ( $\mathrm{CT}_{6}, \mathrm{CT}_{7}, \mathrm{CT}_{8}$, and $\mathrm{CT}_{8}{ }^{\prime}$ ).




It should be noted that the LUMO of the CO fragment has a larger lobe at the C atom than at the O atom, whereas its DOMO has a larger lobe at the O atom. Therefore, $\mathrm{CT}_{8}$ and $\mathrm{CT}_{8}{ }^{\prime}$ are more important than $\mathrm{CT}_{6}$. From the orientation of CO toward CP , it is considered that $\mathrm{CT}_{8}$ is more important than $\mathrm{CT}_{7}$. Consequently, CO is an electron acceptor. This result leads to a prediction that the scheme (4)-II proceeds more easily when the $\mathrm{C}_{2} \mathrm{H}_{4}$ fragment has an electron-donating substituent. This prediction is in line with the experimental result that the reactivity $\left(\mathrm{C}_{3} \mathrm{O}_{2}+\mathrm{C}_{2} \mathrm{H}_{4}\right)$ increases as the number of the methyl groups added to olefin increases. ${ }^{5}$

This reactivity in the scheme (4)-II is explained from another point of view. ${ }^{15}$ The reaction may be initiated by the CT interaction between the ethylenic $\pi$-type bond $\left(\mathrm{C}_{6}-\mathrm{C}_{7}\right)$ and


Figure 10. The summary of the photochemical reaction of $\mathrm{C}_{3} \mathrm{O}_{2}$ with $\mathrm{C}_{2} \mathrm{H}_{4}$ deduced from the present work. $E_{\mathrm{T}}$ is calculated with the SECI method.
the $\sigma$ bond $\left(\mathrm{C}_{1}-\mathrm{C}_{2}\right)$ to be broken. One of the SOMOs of the $\mathrm{C}_{2} \mathrm{O}$ part of the bent CK is localized at the $\mathrm{C}_{1}$ and $\mathrm{C}_{2}$ atoms and has the antibonding nature between them. The mode of the orbital interaction between the HOMO of the ethylenic part and the SOMO of the $\mathrm{C}_{2} \mathrm{O}$ part is depicted in the following figure.


Since the back CT interaction from the SOMO to the LUMO of the ethylenic part does not contribute to the weakening of the $C_{1}-C_{2}$ bond due to the nodal property of the latter, the reaction may be initiated by the ( $\mathrm{HOMO} \rightarrow \mathrm{SOMO}$ ) CT interaction. This CT interaction is accelerated by the introduction of the electron-donating group to the $\mathrm{C}_{2} \mathrm{H}_{4}$ site, which explains the substituent effect on the overall reaction.

It should be noted that the scheme (2)-III apparently gives a favorable pathway according to the orbital interactions.


However, the state correlation diagram shown in Figure $5 \mathrm{in}^{-}$ dicates that the scheme (2)-III is a symmetry-disfavored path for the ${ }^{3} \mathrm{~A}$ state owing to the avoided crossing. Therefore, even if a reaction path is favorable by the mode of the orbital interaction, it becomes unlikely in some excited states. When an excited state is studied, the states of the reacting and product
sites should be correctly correlated. The method of the orbital interaction must be used after they are assigned. As is seen in this figure, in the bent-out-of-plane model the two odd electrons with the same spin are obligated to diffuse into the same region through the orbital interaction. This makes the model unfavorable. It is a well-established characteristic feature of the triplet-state reaction that two odd electrons tend to go away from each other to avoid the disadvantage due to the Pauli exclusion. ${ }^{17}$

## V. Concluding Remarks

In this work, the photochemical reaction of $\mathrm{C}_{3} \mathrm{O}_{2}$ with $\mathrm{C}_{2} \mathrm{H}_{4}$ is discussed by the use of the state correlation diagrams and ab initio calculations. Since $E_{\mathrm{T}}$ 's are calculated with a small basis set (STO-3G), energetics shown in Figures 8 and 9 are merely qualitative. However, some important results for this photochemical reaction are obtained and they are summarized schematically in Figure 10. In this figure, $E_{\mathrm{T}}$ is recalculated by the singly excited configuration interaction (SECI) method to make uniform the energy scale of the overall reaction.

The multistep photochemical reaction of $\mathrm{C}_{3} \mathrm{O}_{2}$ with $\mathrm{C}_{2} \mathrm{H}_{4}$ seems to proceed as follows. The first step is evidently the photodissociation of $\mathrm{C}_{3} \mathrm{O}_{2}$ yielding $\mathrm{C}_{2} \mathrm{O}\left({ }^{3} \Sigma^{-}\right)$and CO . This reaction takes the bent path. The next step is the addition of $\mathrm{C}_{2} \mathrm{O}$ to $\mathrm{C}_{2} \mathrm{H}_{4}$ through the bent-in-plane path, yielding ${ }^{3} \mathrm{CK} .{ }^{18}$ It is decomposed into ${ }^{3} \mathrm{CP}$ and CO through the bent-in-plane path. Since the $E_{\mathrm{T}}$ of ${ }^{3} \mathrm{CP}$ is lower than that of ${ }^{1} \mathrm{CP}$, the ground state $\mathrm{C}_{3} \mathrm{H}_{4}$ may be yielded from ${ }^{3} \mathrm{CP}$ by the isc. The rate-determining step of the overall reaction is found to be the decarbonylation of ${ }^{3} \mathrm{CK}$. Thus, the intermediacy of ${ }^{3} \mathrm{CK}$ and ${ }^{3} \mathrm{CP}$ in the reaction of eq 2 is postulated.

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# An Intermediate Neglect of Differential Overlap Technique for Spectroscopy of Transition-Metal Complexes. Ferrocene 

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#### Abstract

A new intermediate neglect of differential overlap (INDO-SCF-CI) method capable of calculating configuration interaction for transition-metal complexes is described. The technique is characterized by the use of atomic spectroscopic information in the formation of one-center, one-electron matrix elements and in the evaluation of two-electron integrals. All onecentered integrals that mix upon geometric rotations are found to be essential for calculation of configuration interaction and are retained. The method is applied to the calculation of the photoelectric and electronic spectra of ferrocene. Triplet and singlet state $\mathrm{d} \rightarrow \mathrm{d}^{*}$ and charge-transfer transitions are considered. Three nearly degenerate triplet states are calculated for ferrocene at $\sim 20500 \mathrm{~cm}^{-1}$ as compared to an observed triplet state at $18900 \mathrm{~cm}^{-1}$. Three singlet transitions of the $\mathrm{d} \rightarrow \mathrm{d} *$ type are calculated at 21700,23900 , and $31900 \mathrm{~cm}^{-1}$, in very good accord with the experimental observations and assignments of ${ }^{1} \mathrm{E}_{1}{ }^{\prime \prime}$ at $21800 \mathrm{~cm}^{-1},{ }^{1} \mathrm{E}_{2}^{\prime \prime}$ at $24000 \mathrm{~cm}^{-1}$, and ${ }^{1} \mathrm{E}_{1}{ }^{\prime \prime}$ at $30800 \mathrm{~cm}^{-1}$. The lowest charge-transfer excitation is calculated at $36900 \mathrm{~cm}^{-1}$ compared to the experimental observation at $37700 \mathrm{~cm}^{-1}$. A considerable number of states above $36900 \mathrm{~cm}^{-1}$ are calculated and a possible assignment of the observed states in the higher energy region is given. The calculated energies allow spectral assignment in good agreement with experiment and help resolve prior ambiguities in the assignment. The relative energies calculated for the ionic states of the ferroceniun ion ( ${ }^{3} \mathrm{E}_{2}^{\prime}<{ }^{2} \mathrm{~A}_{1}^{\prime}<{ }^{2} \mathrm{E}_{1}{ }^{\prime \prime}$ ) are in good accord with previous ab initio calculations. The first two of these states are formed from the formal loss of an electron from a metal orbital; the third, from the loss of an electron from a ligand orbital. For all of these states relaxation upon ionization is so significant that the net charge on iron increases only negligibly from +1.9 in ferrocene to +2.0 in the ferrocene ion. This observation is also in agreement with $a b$ initio findings and has some support from Mössbauer spectroscopy.


## I. Introduction

To calculate the properties of organometallic transitionmetal complexes from molecular orbital theory is often a difficult task. The number of integrals to be evaluated in an ab initio technique increases rapidly when a transition-metal atom is included in the molecule of interest. Care must be taken to treat in a consistent fashion the metal atoms and the organic moieties of the molecule. Extensive calculations often must be
performed in order to explain differences between the normally localized metal orbitals and the typically delocalized ligand orbitals. It becomes of great interest therefore to develop a semiempirical model which can easily and successfully calculate properties of transition-metal complexes.

The electronic structure of transition-metal complexes has been described most frequently in terms of ligand field theory. ${ }^{2 a}$ Of great qualitative value, this approach is usually used


[^0]:    ${ }^{a}$ See Figure 4, ${ }^{b}$ See Figure 5.

